Formal Symbolic Logic — The Argument in Modern Notation

1. Predicates Defined

Predicate	Meaning
T(x)	x is transgender
M(x)	x is mentally ill
W(x)	x is a woman
S(x)	x is a man (male / of the male sex)
P(y)	y is a women's single-sex space
E(x, y)	x enters y
Wr(x)	x acts wrongly

2. The Five Statements Formalised

P1 — ALL transgender people are mentally ill

$$\forall x \big(T(x) \rightarrow M(x) \big)$$

For all x: if x is transgender, then x is mentally ill.

In set terms: the set of all transgender people is a subset of the set of mentally ill people.

$$T \subseteq M$$

P2 — ALL transgender women ARE men

$$\forall x \big(T(x) \wedge W(x) \rightarrow S(x) \big)$$

For all x: if x is transgender and x is a woman, then x is a man.

In set terms: the set of all transgender women is a subset of the set of men.

$${x : T(x) \wedge W(x)} \subseteq S$$

P3 — Men entering women's same-sex spaces is wrong

$$\forall x \forall y \big(S(x) \wedge P(y) \wedge E(x, y) \rightarrow Wr(x) \big)$$

For all x and for all y: if x is a man, and y is a women's single-sex space, and x enters y, then x acts *wrongly.*

P4 — Mentally ill men emphatically should not enter women's single-sex spaces

$$\forall x \forall y \big(S(x) \wedge M(x) \wedge P(y) \wedge E(x, y) \rightarrow Wr(x) \big)$$

For all x and for all y: if x is a man, and x is mentally ill, and y is a women's single-sex space, and x enters y, then x acts wrongly.

> ***Note:*** *This premise contains its own problem — it forbids all mentally ill men from entering women's spaces. If P1 is accepted (transgender = mentally ill), then P4 categorically excludes all transgender people from women's spaces regardless of any other consideration.*

Conclusion — Therefore: transgender women should not enter single-sex spaces

$$\forall x \forall y \big(T(x) \wedge W(x) \wedge P(y) \wedge E(x, y) \rightarrow Wr(x) \big)$$

For all x and for all y: if x is transgender and x is a woman, and y is a women's single-sex space, and x enters y, then x acts wrongly.

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3. Proof That the Conclusion Follows Validly from the Premises

We demonstrate that the conclusion is a **logical consequence** of P1–P4.

Step 1: From P1 and P2, derive the properties of a transgender woman

Let x be arbitrary.

- From **P2**: if $T(x) \wedge W(x)$ then $S(x)$.
- From **P1**: if $T(x)$ then $M(x)$.
- Therefore, for any x that is $T(x) \wedge W(x)$, we have: **$S(x) \wedge M(x)$**.
  (x is both a man and mentally ill)

Formally, by conditional proof within $\forall x$:

$$\forall x \big(T(x) \wedge W(x) \rightarrow S(x) \wedge M(x) \big)$$

This follows directly from P1 and P2 by the transitivity of $\rightarrow$ (modus ponens applied to the antecedent $T(x) \wedge W(x)$).

Step 2: From the derived property, apply P4

Let x be arbitrary and let y be a women's space P(y).

- From **Step 1**: for any x with $T(x) \wedge W(x)$, we have $S(x) \wedge M(x)$.
- From **P4**: for any x with $S(x) \wedge M(x)$, and any y with $P(y)$, if $E(x,y)$ then $Wr(x)$.

Combine these:

$$\forall x \forall y \big(T(x) \wedge W(x) \wedge P(y) \wedge E(x, y) \rightarrow Wr(x) \big)$$

This is the conclusion.

∎ (QED)

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4. The Full Valid Derivation

P1	$\forall x (T(x) \to M(x))$	*All transgender people are mentally ill*
P2	$\forall x (T(x) \wedge W(x) \to S(x))$	*All transgender women are men*
P3	$\forall x \forall y (S(x) \wedge P(y) \wedge E(x,y) \to Wr(x))$	*Men entering women's spaces is wrong*
P4	$\forall x \forall y (S(x) \wedge M(x) \wedge P(y) \wedge E(x,y) \to Wr(x))$	*Mentally ill men should not enter women's spaces*
C	$\forall x \forall y (T(x) \wedge W(x) \wedge P(y) \wedge E(x,y) \to Wr(x))$	*Transgender women should not enter women's spaces*

Derivation:

1. P2: $\forall x (T(x) \wedge W(x) \to S(x))$ ………….. *Premise*
2. P1: $\forall x (T(x) \to M(x))$ …………………. *Premise*
3. From 1 + 2 (conjunction of antecedents):
   $\forall x (T(x) \wedge W(x) \to S(x) \wedge M(x))$ ……….. *Derived*
4. P4: $\forall x \forall y (S(x) \wedge M(x) \wedge P(y) \wedge E(x,y) \to Wr(x))$ …….. *Premise*
5. Universal instantiation, x and y arbitrary:
   $(T(a) \wedge W(a) \wedge P(b) \wedge E(a,b)) \to (S(a) \wedge M(a) \wedge P(b) \wedge E(a,b))$ …….. *From 3*
6. $S(a) \wedge M(a) \wedge P(b) \wedge E(a,b) \to Wr(a)$ …………. *From 4*
7. $(T(a) \wedge W(a) \wedge P(b) \wedge E(a,b)) \to Wr(a)$ ………. *From 5 + 6, hypothetical syllogism*
8. Re-generalise: $\forall x \forall y (T(x) \wedge W(x) \wedge P(y) \wedge E(x,y) \to Wr(x))$ ……….. *From 7, universal generalisation*

∴ C follows necessarily from P1–P4. The argument form is **valid.**

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5. Summary Truth Table — What the Form Guarantees

Premise combination	Conclusion	What this means
P1 ∧ P2 ∧ P3 ∧ P4 all TRUE	C TRUE	Sound argument
P1 FALSE (transgender ≠ mentally ill)	C has no guaranteed truth	**Argument collapses**
P2 FALSE (transgender women ≠ men)	C has no guaranteed truth	**Argument collapses**
P3 FALSE (men in women's spaces ≠ wrong)	C has no guaranteed truth	Argument collapses
P4 FALSE (mentally ill men in women's spaces ≠ wrong)	C has no guaranteed truth	Argument collapses

Since P1 and P2 are demonstrably false (see companion document), the argument is valid but unsound — its truth value is null.

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6. The Hidden Assumption — Exposed

The entire argument rests on one unstated but essential premise:

> ***"Gender identity is determined by biological sex, and social/legal recognition of gender identity is irrelevant to the definition of 'man' and 'woman.'"***

Formalised:

$$\forall x \big(W(x) \leftrightarrow F(x) \big) \wedge \forall x \big(M(x) \leftrightarrow S(x) \big)$$

Where F(x) = x is female-sexed and S(x) = x is male-sexed.

This is the axiom that underlies P2. But it is itself philosophically contested, empirically disputed, and — most importantly — **it is precisely what the argument is trying to prove, not a premise from which the conclusion follows.**

This is the **question-begging** (petitio principii) at the heart of the argument.

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